Technical Notes

TECHNICAL NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed 6 manuscript pages and 3 figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Free Vibration of Curved Skew Panels

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Introduction

ONE of the approximate ways of analyzing the wings of the aircraft is to treat them as skew plates. Since the wings are curved, it will be more appropriate to consider them as curved skew panels. The free vibration analysis of elastic cylindrical panels has been done using classical methods in Ref. 1, for example, and adopting the finite strip method in Refs. 2 and 3. But there is no literature available for the vibration analysis of skew shell panels.

The skew panel analyzed here is defined as the surface obtained by moving a straight line generator over an arc of a circle (with curvature R), whose plane is not at right angles to the generator. The panel is a parallelogram in plan (see Fig. 1). Using higher-order finite strip method[4] and shallow shell theory in oblique coordinates,[5,6] the title problem is analyzed.

Basic Equations

The stresses in oblique coordinates can be defined as the corresponding stress resultant forces per unit cross-sectional area measured perpendicular to the direction of the so-called direct stresses. Then the membrane stresses in rectangular and oblique coordinates (see Fig. 1) are related[5] as

$$\{\bar{\sigma}\} = [A]\{\sigma\} \tag{1}$$

If the constitutive equation for elastic orthotropic material subjected to plane stress is

$$\{\bar{\sigma}\} = [\bar{E}]\{\bar{\epsilon}\} \tag{2}$$

then the stress-strain relation in oblique coordinates is

$$\{\sigma\} = [A]^{-1}[\bar{E}]\,[A^T]^{-1}\{\epsilon\} = [E]\{\epsilon\} \tag{3}$$

Following Ref. 6, the expressions for strain energy and kinetic energy (after neglecting inplane inertia terms) for the shell are

$$U = (h/2)\sin\alpha \iint [\{\epsilon\}^T[E]\{\epsilon\} + (h^2/12)\{\chi\}^T[E]\{\chi\}]\,dx\,dy \tag{4}$$

$$T = (\rho h\omega^2/2)\sin\alpha \iint w^2 dx\,dy \tag{5}$$

where $\{\chi\}$ is the curvature vector, h is the thickness of the shell, ρ is mass density, and ω is the angular frequency.

The shell panel is assumed to be supported on shear diaphragms on the edges parallel to the x axis. It is divided into a number of strips. The displacement function for a typical strip (Fig. 1) satisfying the kinematic boundary conditions in the y direction can be taken as

$$u = \sum_{n=1}^{N}\sum_{i=1}^{4} (Au)_{ni} x^{m_i} \sin\frac{n\pi y}{\ell} \tag{6a}$$

$$v = \sum_{n=1}^{N}\sum_{i=5}^{8} (Av)_{ni} x^{p_i} \cos\frac{n\pi y}{\ell} \tag{6b}$$

$$w = \sum_{n=1}^{N}\sum_{i=9}^{14} (Aw)_{ni} x^{r_i} \sin\frac{n\pi y}{\ell} \tag{6c}$$

where $m_i = 0,1,2,3$ for $i = 1,2,3,4$; $p_i = 0,1,2,3$ for $i = 5,6,7,8$; and $r_i = 0,1,2,3,4,5$ for $i = 9,10,11,12,13,14$. These coefficients are not defined outside of these intervals.

The coefficients Au, Av, Aw can be written in terms of the displacement parameters u, u_x, v, v_x, w, w_x, w_{xx} at the ith and jth edges of the typical strip as

$$\{Au\}_n = [Tu]\{\delta u\}_n,\ \{Av\}_n = [Tv]\{\delta v\}_n,\ \{Aw\}_n = [Tw]\{\delta w\}_n$$

The matrices $[Tu]$, $[Tv]$, and $[Tw]$ are given in the Appendix. These can be combined and written as

$$\{A\}_n = [T]\{\delta\}_n \tag{7}$$

where $\{\delta u\}^T = [u_i,\ u_{x,i},\ u_j,\ u_{x,j}]$, etc., and the suffix n represents the parameter corresponding to the nth harmonic. Combining Eqs. (4) and (6),

$$U = \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N}\{A\}_m^T[\hat{K}]_{mn}\{A\}_n$$

using Eq. (7)

$$U = \frac{1}{2}\sum_{m=1}^{N}\sum_{n=1}^{N}\{\delta\}_m^T[T]^T[\hat{K}]_{mn}[T]\{\delta\}_n$$

$$\begin{aligned}(\hat{K}_{ij})_{mn} = \langle\, & [E_{11}\{m_i m_j F(m_i+m_j-1) + R^2F(r_i+r_j+1) \\ & -R[m_jF(r_i+m_j)+m_iF(r_j+m_i)]\}\} \\ & +E_{22}k_n^2F(p_i+p_j+1) \\ & +E_{33}\{k_n^2F(m_i+m_j+1) \\ & +p_ip_jF(p_i+p_j-1)+k_n[p_jF(m_i \\ & +p_j)+p_iF(m_j+p_i)]\}\}\end{aligned}$$

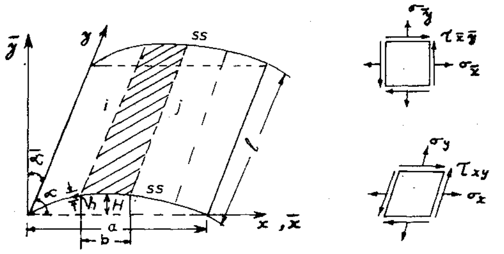

Fig. 1 Skew cylindrical panel.

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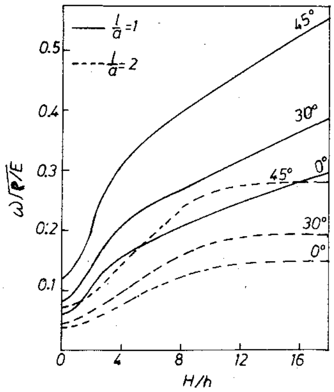

Fig. 2 Variation of fundamental frequency parameter with H/h.

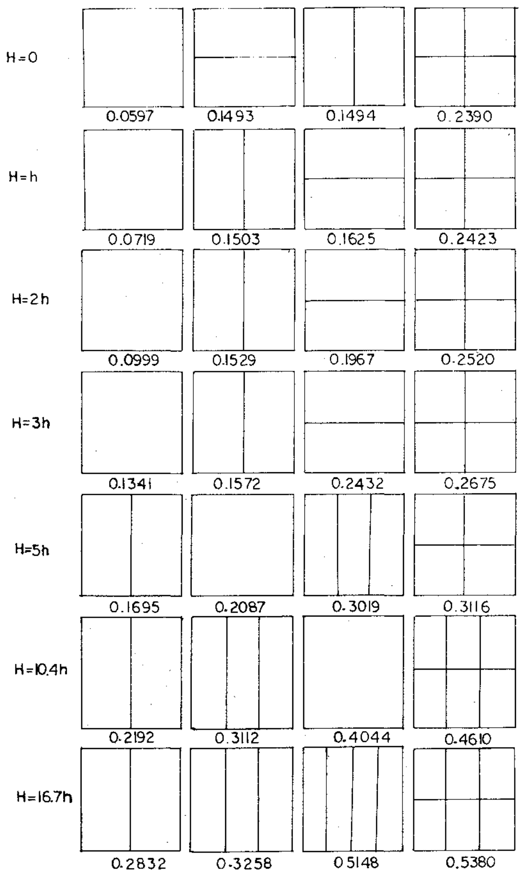

Fig. 3 Mode shapes for rectangular curved panel for $\bar{\alpha} = 0°$.

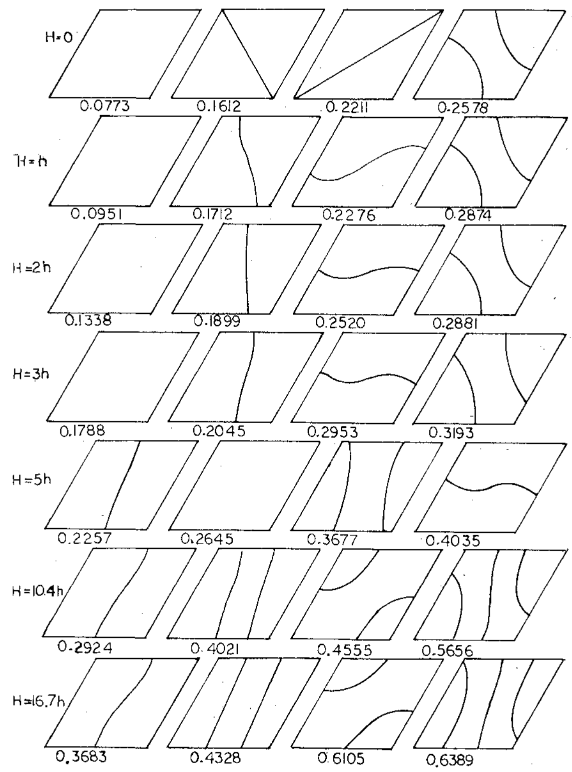

Fig. 4 Mode shape for skew curved panel for $\bar{\alpha} = 30°$.

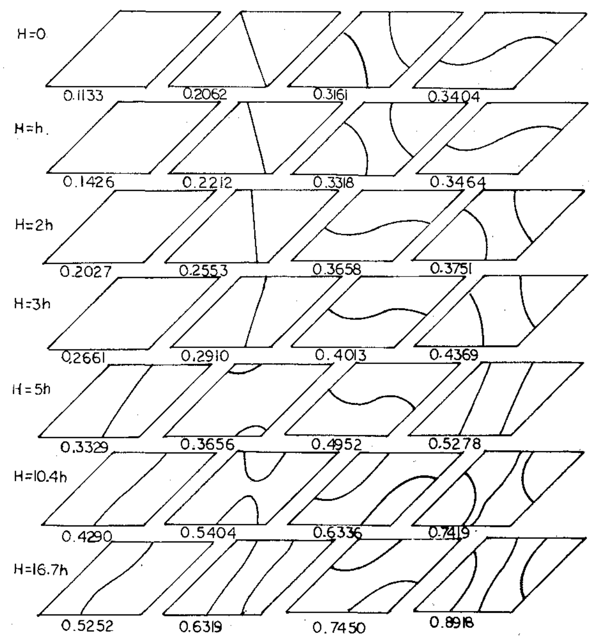

Fig. 5 Mode shape for skew curved panel for $\bar{\alpha} = 45°$.

$$-E_{12}k_n\{m_iF(m_i+p_j)+m_jF(m_j+p_i)$$
$$-R[F(r_i+p_j+1)+F(r_j$$
$$+p_i+1)]\}](\ell/2)\delta_{mn}$$
$$+E_{13}\{(k_nm_iI_{mn,}+k_mm_jI_{n,m})F(m_i+m_j)$$
$$+m_ip_jF(m_i+p_j-1)I_{m,n}$$
$$+m_jp_iF(m_j+p_i-1)I_{n,m}$$
$$-R[k_nF(r_i+m_j+1)I_{m,n}$$
$$+k_mF(r_j+m_i+1)I_{n,m}$$
$$+p_jF(r_i+p_j)I_{m,n}+p_iF(r_j+p_i)I_{n,m}]\}$$
$$-E_{23}\{k_mk_n[F(m_i+p_j+1)I_{n,m}$$
$$+F(m_j+p_i+1)I_{m,n}]$$
$$+(k_mp_jI_{m,n}+k_np_iI_{n,m})F(p_i+p_j)\}\rangle h\sin\alpha$$
$$+\langle\{E_{11}r_ir_j(r_i-1)(r_j-1)F(r_i$$
$$+r_j-3)+E_{22}k_n^4F(r_i+r_j+1)$$
$$+4E_{33}k_n^2r_ir_jF(r_i+r_j-1)$$

Table 1 Convergence study of skew shell panel ($\bar{\alpha}=45°$, $h/a=0.01$, $H/h=10.16$, $\ell=a=1$, $\nu=0.3$)

No of harmonics	Mode	$\omega(\rho/E)^{1/2\,a}$ No of strips 2	3	4
4	1	0.079	0.078	0.077
4	2	0.107	0.100	0.099
4	3	0.137	0.117	0.116
4	4	0.145	0.137	0.134
6	1	0.079	0.077	0.077
6	2	0.103	0.097	0.096
6	3	0.136	0.115	0.114
6	4	0.143	0.134	0.132

[a] Has a dimension of 1/length.

$$-E_{12}k_n^2[r_i(r_i-1)$$
$$+r_j(r_j-1)]F(r_i+r_j-1\}(\ell/2)\,\delta_{mn}$$
$$+\{2E_{13}r_ir_j[k_n(r_i-1)I_{m,n}$$
$$+k_m(r_j-1)I_{n,m}]F(r_i+r_j-2)$$
$$+2E_{23}k_mk_n(k_mr_jI_{m,n}$$
$$+k_nr_iI_{n,m})F(r_i+r_j)\}\rangle(h^3/12)\sin\alpha$$

where $F(m)=b^m/m$; $k_m=m\pi/\ell$; δ_{mn} is the Kronecker delta; and

$$I_{m,n}=\int_0^\ell \sin\left(\frac{m\pi y}{\ell}\right)\cos\left(\frac{n\pi y}{\ell}\right)dy$$

Then the stiffness matrix $[K]$ consists of $N\times N$ submatrices, typical of which is $[K_{mn}]=[T]^T[\hat{K}]_{mn}[T]$.

Similarly, combining Eqs. (5) and (6), the mass matrix $[M]$ is given by diagonal submatrices such as $[M]_{11}$, $[M]_{22}\cdots$, $[M]_{NN}$, in which $[M]_{nn}=[Tw]^T[\hat{M}]_{nn}\,[Tw]$ and $(\hat{M}_{ij})_{nn}=(\ell/2)\;(b^{r_i+r_j+1})/(r_i+r_j+1)$. After assembling the stiffness and mass matrices, the geometrical boundary conditions in the x direction are introduced, and then this eigenvalue problem is solved.

Numerical Work and Disucssion

The numerical work has been done for an isotropic panel supported on shear diaphragms on all edges. Convergence of frequency has been studied by taking different numbers of harmonics and also increasing the number of strips (see Table 1), The convergence of the solution is good. Further calculations have been done using four strips and six harmonics. The frequencies for the particular cases (viz. curved rectangular panel and flat skew plate) were compared (not reported here) with those of Sewall[1] and Durvasula,[7] respectively, and they were found to agree well.

The influence of rise and skew angle on the fundamental frequency parameter $[\omega(\rho/E)^{1/2}]$, which has the dimension of 1/length, is given beneath each nodal pattern.

As the rise of the panel is increased, the membrance action comes into play, and this is evidenced by the rapid increase in the frequency of the nodeless mode (see Fig. 3). When $H/h>5$, the lowest mode for all of the skew angles considered has a single nodal line. For this mode shape, the energy will be equally due to bending and stretching, whereas for higher modes with more than one vertically oriented node the energy will be predominently due to bending. For the skewed panels at $H/h\geq 5$, explicit distinction between membrane and bending effects is obscured by the disappearance of the nodeless modes and the existence of highly curved modes. By choosing proper functions in the y direction, shells with different types of boundary conditions can be analyzed.

Appendix

$$[Tu]=[Tv]=\begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 \\ -3/B^2 & -2/B & 3/B^2 & -1/B \\ 2/B^3 & 1/B^2 & -2/B^3 & 1/B^2 \end{bmatrix}$$

$$[Tw]=\begin{bmatrix} 1.0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 & 0 \\ -10/B^3 & -6/B^2 & -1.5/B & 10/B^3 & -4/B^2 & 0.5/B \\ 15/B^4 & 8/B^3 & 1.5/B^2 & -15/B^4 & 7/B^3 & -1/B^2 \\ -6/B^5 & -3/B^4 & -0.5/B^3 & 6/B^5 & -3/B^4 & 0.5/B^3 \end{bmatrix}$$

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Consistent Mass Matrix in Fluid Sloshing Problems

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SEVERAL recent publications have presented a technique for the solution of problems involving the free-surface vibration of fluids in a container by a finite element method.[1-4] In these works Lagrange's equation is used to formulate a standard matrix vibration equation using the displacement formulation common to most finite element structural analysis.

In this present Note it is shown that a derivation of the mass matrix for such sloshing problems based on lumping the effective vibration mass at various nodes gives much inferior results in comparison to a consistent mass matrix. The consistent mass matrix is of the same bandwidth as the stiffness matrix whereas the lumped mass approach has only the diagonal terms nonzero. Thus, a consistent mass formulation implies a time penalty on computations.

With structural and elasticity vibration problems, there is much evidence[5,6] (and many more) that there is only a

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